On ideals of rings of continuous integer-valued functions on a frame

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Let L be a zero-dimensional frame and $\mathfrak{Z}L$ be the ring of integer-valued continuous functions on L. We associate with each sublocale of ζL, the Banaschewski compactification of L, an ideal of $\mathfrak{Z}L$, and show the behaviour of these types of ideals. The socle of $\mathfrak{Z}L$ is shown to be always the zero ideal, in contrast with the fact that the socle of the ring $\mathcal{R}L$ of continuous real-valued functions on L is not necessarily the zero ideal. The ring $\mathfrak{Z}L$ has been shown by B. Banaschewski to be (isomorphic to) a subring of $\mathcal{R}L$, so that the ideals of the larger ring can be contracted to the smaller one. We show that the contraction of the socle of $\mathcal{R}L$ to $\mathfrak{Z}L$ is the ideal of $\mathfrak{Z}L$ associated with the join (in the coframe of sublocales of ζL) of all nowhere dense sublocales of ζL. It also appears in other guises.