Master of Science Exam (Presentation)

Date: Monday, April 11, 2022

Time: 1:00 pm

Location: SE 271

Speaker: Raya Shimshi (in person)

Title: Super Convergence of Ergodic Averages for Quasiperiodic Orbits (Quantitative Quasiperiodicity)

Abstract: The Birkhoff Ergodic Theorem asserts that time averages, that is, Birkhoff averages, Σ f(x\_n)/N, of a function f evaluated along an ergodic trajectory (x\_n) of a function T of length N converge to the space average ∫ f dµ, where µ is the unique invariant probability measure. But that convergence can be slow. We introduce a modified average of f(x\_n).  Instead of uniform averages that assign equal weights to points along the trajectory, we use an average with a non-uniform distribution of weights, weighing the early and late points of the trajectory much less than those near the midpoint N/2.  When (x\_n) is a trajectory on a quasiperiodic torus and f and T are C infinity, we show that our weighted Birkhoff averages converge “super” fast to ∫ f dµ, i.e. with error smaller than every polynomial of 1/N. Our goal is to show that the Weighted Birkhoff Average is a powerful computational tool. This result can be applied to obtain efficient numerical computation of rotation numbers, conjugates (i.e. changes of variables) and their Fourier series of quasiperiodic systems, often with 30-digit precision.