MATH CIRCLE AT FAU

MORE COUNTING, THIS AND THAT
Ms. Nakamura’s class has 25 students. Ms. Nakamura wants to select 7 students to represent the class at the next festivities. In how many different ways can she do this?
A QUICK REFRESHER

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\[
\begin{align*}
n! &= 1 \times 2 \times 3 \times \cdots \times n \\
1! &= 1, \ 2! = 2, \ 3! = 6, \ 4! = 24, \ 5! = 120, \ldots \\
0! &= 1 \quad \text{(Because it works)} \\
\binom{n}{k} &= \frac{n(n-1)\cdots(n-k+1)}{k!},
\end{align*}
\]

\[
\begin{array}{cccccccc}
\binom{n}{0} &= 1 \left( \frac{n}{1} \right) &= n, & \binom{n}{1} = \frac{n(n-1)}{2}, & \binom{n}{2} = \frac{n(n-1)}{2}, & \binom{n}{3} = \frac{n(n-1)(n-2)}{6}, & \ldots, & \binom{n}{n} = 1.
\end{array}
\]

\[
\binom{n}{k} = 0 \quad \text{if} \quad k > n.
\]
Ms. Nakamura’s class has 25 students, 10 boys and 15 girls. Of the students, 4 boys and 7 girls are excellent singers; the rest of the students are just so-so. Ms. Nakamura has to assemble a cast for a production of an opera. She needs 2 boys and 3 girls with excellent voices for the lead roles, and then a chorus of 5 boys and 5 girls from among the remaining students, making sure that the excellent singers not chosen for the lead roles are part of the chorus. In how many different ways can such a cast be assembled?
Two people left at dawn, at the exact same time, one traveling from A to B, the other one from B to A. They travel at a constant speed, without stopping. They meet at noon. The first one arrives at B at 4 p.m., the second one arrives at A at 9 p.m.

At what time was dawn that day?
THE 2, 3, 5 QUESTION

- How many numbers in the range 1-1000 are NOT divisible by 2, 3, or 5?
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A TILING PROBLEM

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Once you figure out the answer, here are two additional challenges: What is the answer if the strip to be tiled is 15'' wide? What if the tiles are of different colors? Say black and white. (To think about at home)