MATH CIRCLE AT FAU
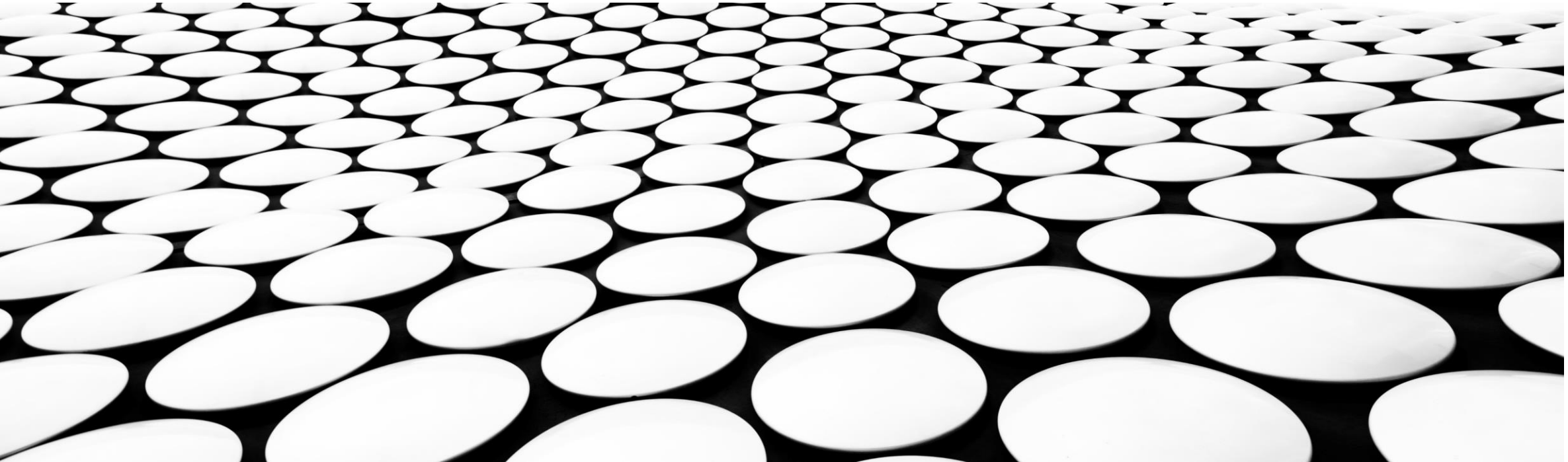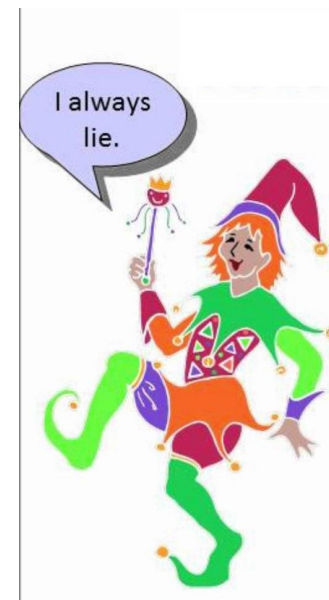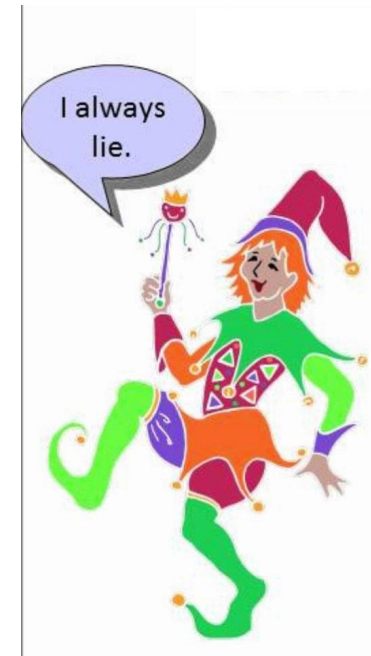
KNIGHTS AND KNAVES – IMPLICATIONS.

- We are back in the isle of knights and knaves. We meet four natives, Al, Belinda, Carl and Dorothy. Al is wearing a hat and when we ask him ``are you knight or knave?'' he replies:

- *If I am a knight, then I will eat my hat.*

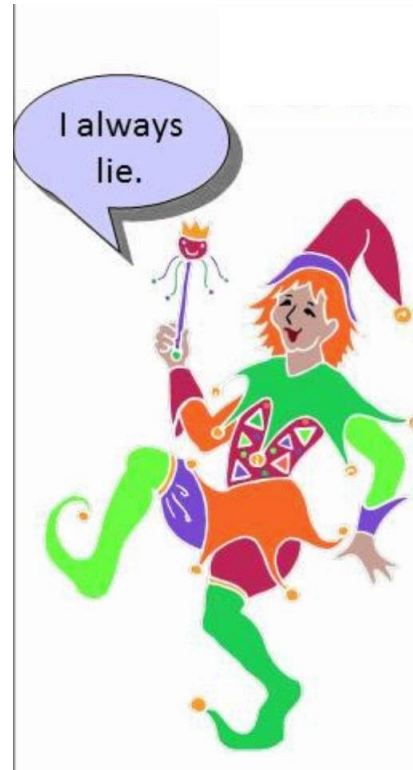- Prove that Al MUST eat his hat. Can All be a knave? Why? Why not?

KNIGHTS AND KNAVES – IMPLICATIONS.

- Belinda now answers the same question with:

- *If I am a knave, then $2 + 2 = 4$.*

- What is Belinda?

KNIGHTS AND KNAVES – IMPLICATIONS.

- Carl now says:

- *If Dorothy is a knight, then I am a knave.*

- What are Carl and Dorothy?

PEGGING ALONG

- Five (5) yellow pegs, four (4) red pegs, three (3) green pegs, two (2) blue pegs and one (1) orange peg are to be placed on a triangular peg board, like the one pictured below. In how many different ways can this be done if no horizontal row or vertical column is to contain two pegs of the same color.
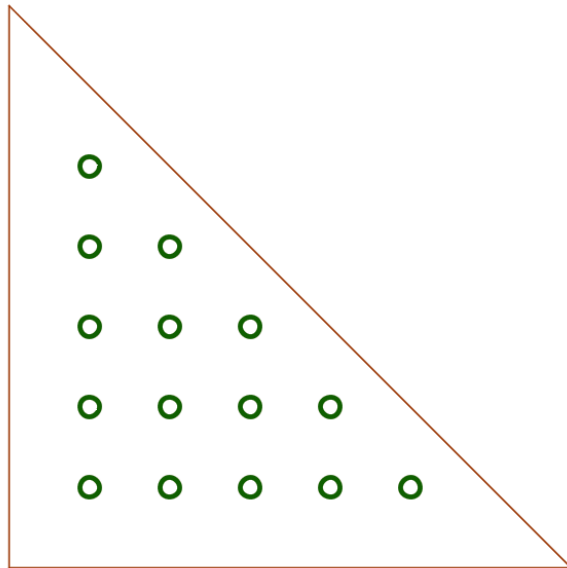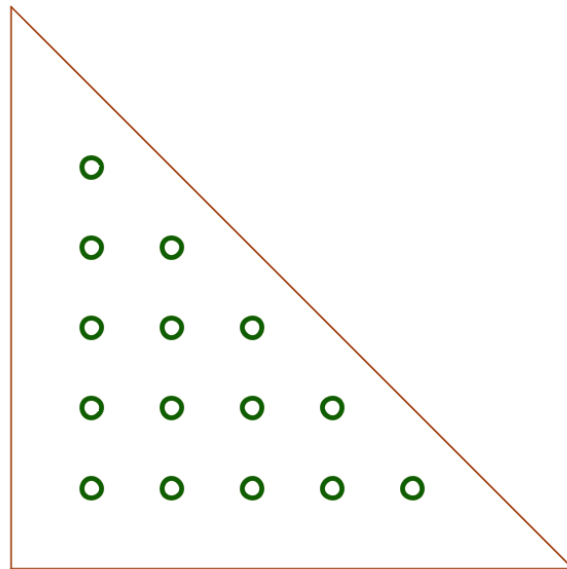
PEGGING ALONG

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Same question if we relax the conditions, allow the same color in some columns, but no horizontal row is to contain two pegs of the same color.

SHADING THE GRID
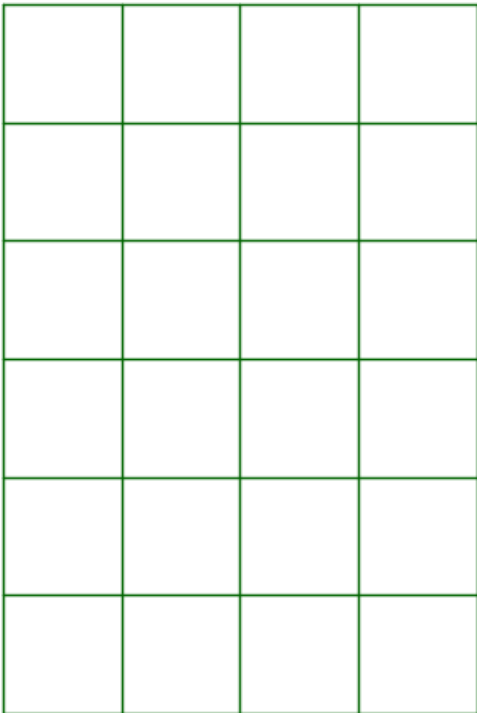
One wishes to shade 12 squares of the grid on the left in such a way that every column has exactly three shaded squares and every row has exactly two shaded squares.
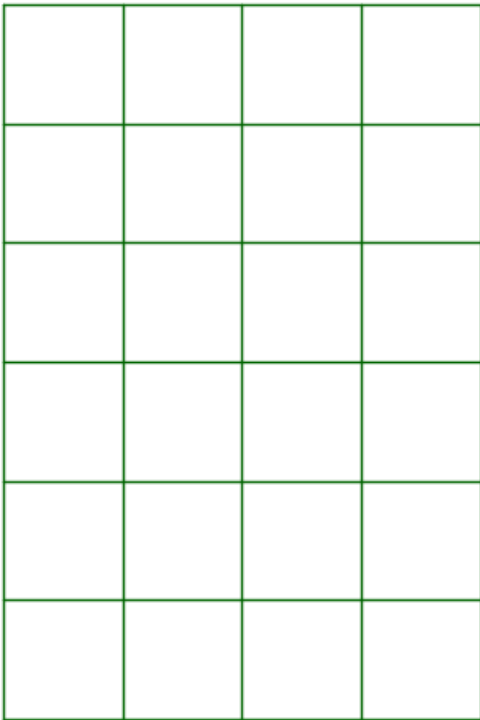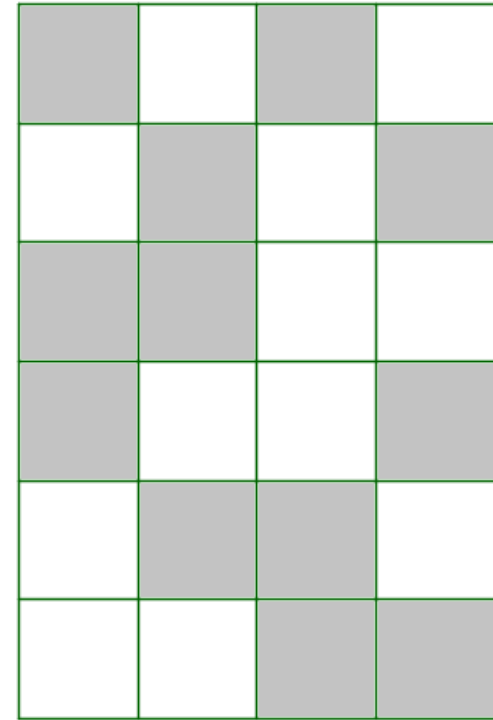
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SHADING THE GRID

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In how many ways can one do this?

Here is one way.

MATH CLUBS

- Of the students attending a high school, 60% are girls and 45% of all the students are members of the Math Club. One day, the school gets 20 additional students, all of them boys and 19 of them join the math club.

- Now 55% of the students are girls (of course, the total number of girls has not changed, just their percentage).

- How many students are now members of the math club?
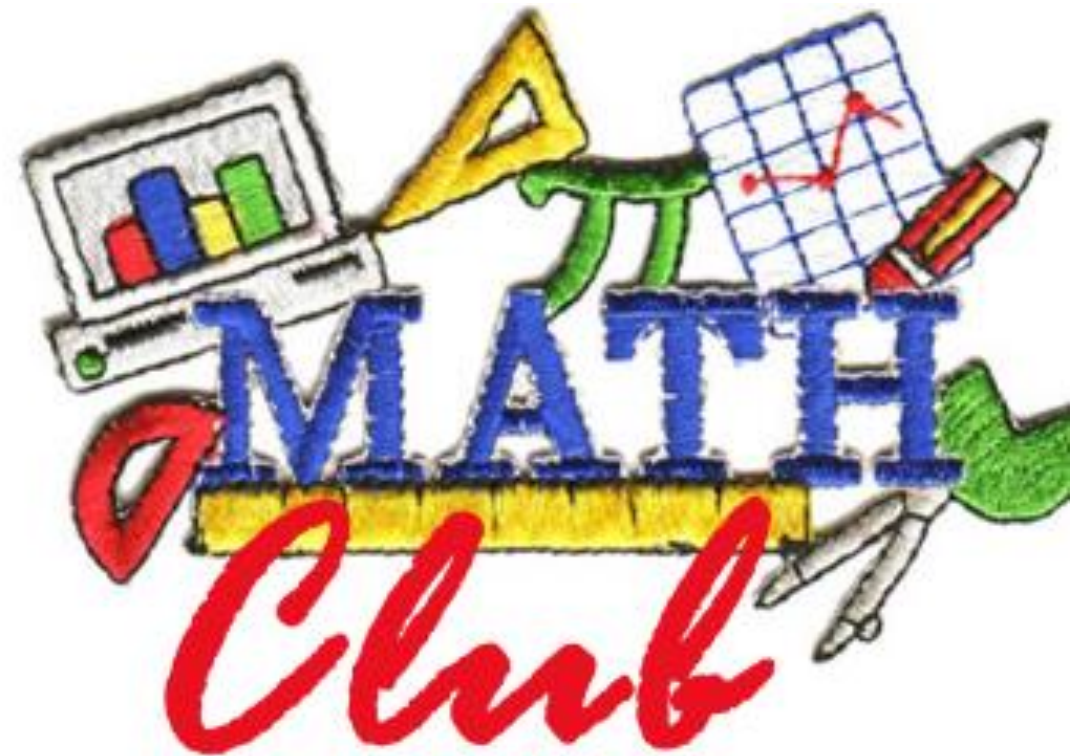
ELEVEN RULES

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- For example, 25531 has quotient 255, remainder 31, when divided by 100. Now

$$255 + 31 = 286 = 26 \times 11$$
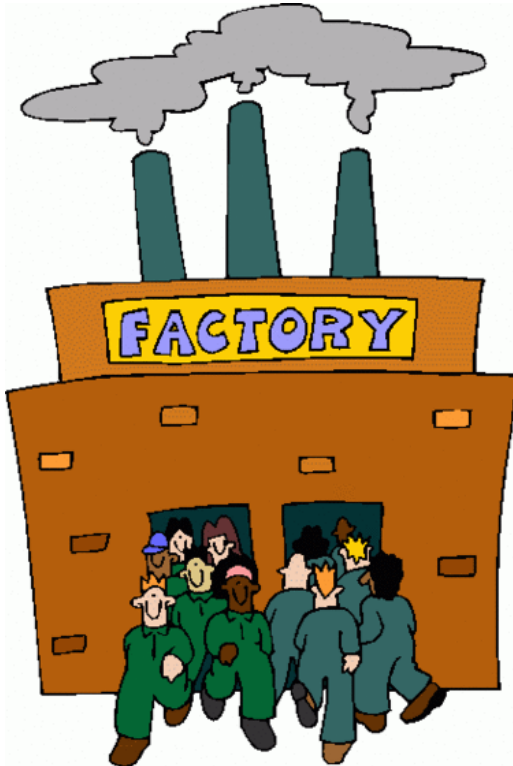
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- How many elevenish numbers are there?

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So 25531 is elevenish.

TRICKY TRIMBLES

- A factory making trimbles has 10 workers; the chief, an experienced older craftsperson, and 9 recent young graduates from NTU (Northeastern Trimble University). Each of the nine young workers produces 15 trimbles a day while the chief produces 9 more trimbles per day than the average of all 10 workers.

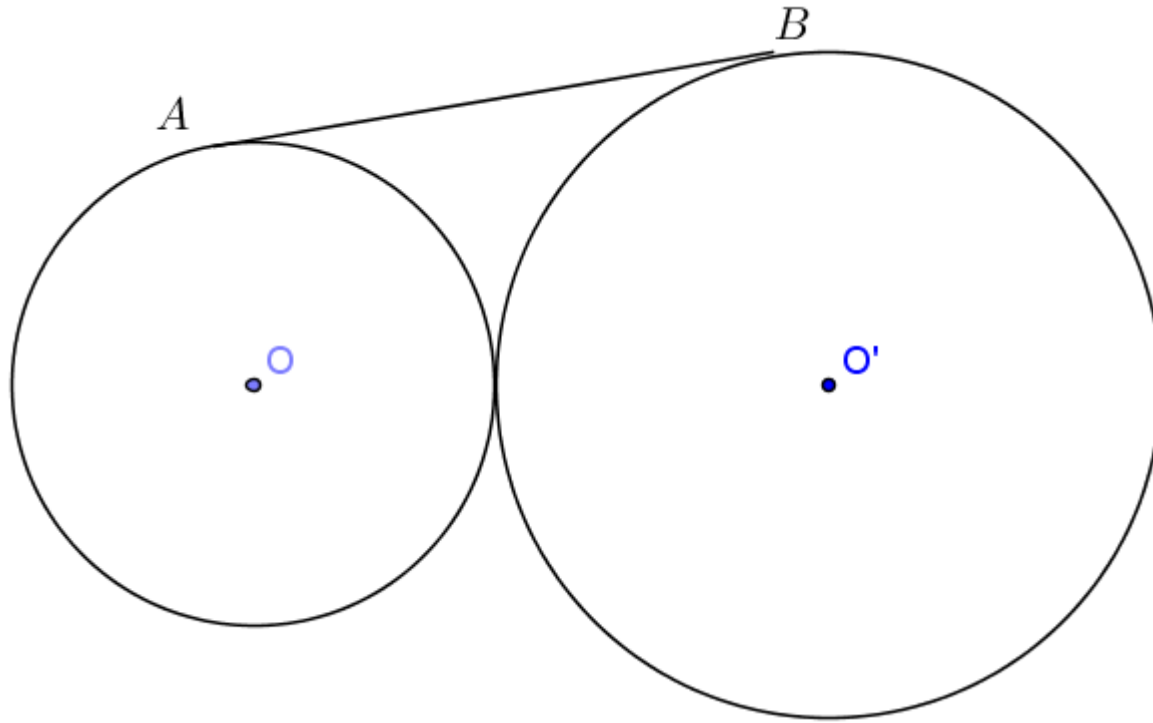- How many trimbles are produced in a day?

ORNAMENTATION

A metal shop has 36 plates of metal to produce metal ornaments. It takes one plate to produce one ornament.

However, in producing the ornaments some metal is left over. The metal left over from 6 plates can be molten and made into another plate. How many ornaments can be made in all?
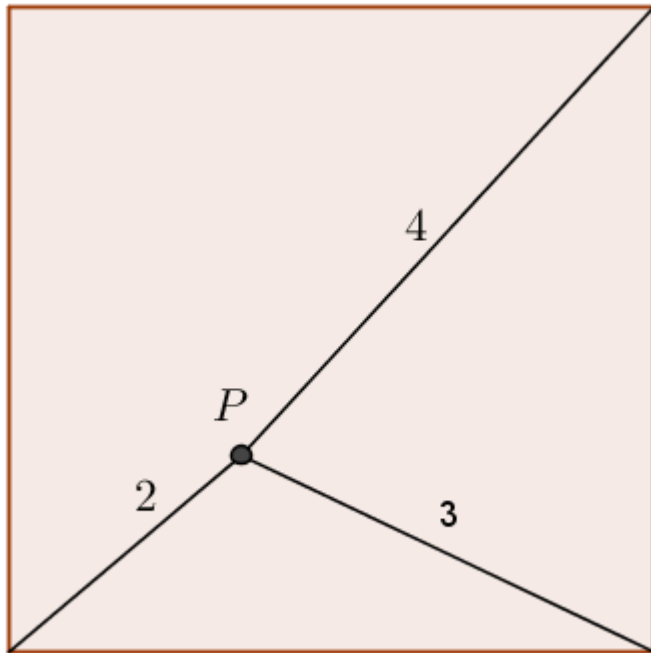
CIRCULATING

The circles at centers O;O0 are tangent to each other. The segment AB is tangent to both circles. If the radius of the circle of center O is 4, the radius of the other circle is 6, what is the length of the segment AB?

ALL PRAISE PYTHAGORAS

A point P inside a square is at distance 2 units from one vertex of the square, at distance 3 units from the next and at distance 4 units from the following one, as shown in the figure to the left. What is the area of the square?

As a hint, there will be a square root in the correct solution.