MATH CIRCLE AT FAU

JANUARY 27, 2024



CHESS AND DOMINOES

• This is a classic. You may have seen it before.



Can one cover by non-overlapping dominoes an 8×8 chessboard with two opposite corners removed.

Each domino can cover exactly two adjacent chessboard squares, so you would need 31 dominoes.

Chessboard, two coners removed

A SMALL PARTY

• There are 8 people at a party. Show that it is possible for each of them to shake hands with exactly 3 other people.



ANOTHER SMALL PARTY

There are 9 people at a party. Show that it is impossible for each of them to shake hands with exactly 3 other people.



A CRAZY TEACHER

A class of 25 students has their desks arranged in a 5 × 5 square. No more than one student can fit at a single desk. One morning, with all the students seated, the teacher asks every student to move to an adjacent desk (directly in front, behind, to the left, or to the right of their current desk). Can this be done?

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- Can you predict what that number will be? Is it always the same?

ERASING NUMBERS

The numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 are written on the board. A person can pick two numbers a and b from the board, erase them and replace them with the number a + b - 1.

- (a) How many times can one repeat this process until there is just one number on the board?
- (b) Can you say ahead of time what this number will be?