Fashion game on graphs

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In this paper, we propose and study an optimization problem of the fashion game on graphs, which can be regarded as a graph extension of matching pennies. There are two kinds of players in a graph G: Conformists and Rebels. All players choose their actions from an identical set of the two symmetric actions, say $\{0, 1\}$. An action profile π of G is a mapping from the vertex set of G to the action set $\{0, 1\}$. A conformist (resp. rebel) likes people having the same (resp. different) action with her and dislikes people having the different (resp. same) action. The utility $u(v, \pi)$ of a player v under the action profile π is the number of neighbors she likes minus the number of neighbors she dislikes. The utility $u(G, \pi)$ of G under π is the minimum utility among all players. Let t be an integer. A graph G is said to be t-satisfiable if there is an action profile of G such that all players have utilities at least t. The utility of G, denoted by $u(G)$, is the maximum t such that G is t-satisfiable.
We provide simple characterizations to determine the utilities of paths, cycles, and complete graphs. We design a linear-time algorithm to determine the utility of a tree. We obtain lower bounds of utilities of graphs containing only rebels, t-degenerate graphs, and the kth power of paths, respectively. We also prove that for any fixed integer $t \geq -2$, the problem of deciding if a graph containing both conformists and rebels is t-satisfiable is NP-complete, and for any fixed integer $t \geq 1$, the problem of deciding if a graph containing only rebels is t-satisfiable is also NP-complete. We finally propose some further research problems on this topic.

Keywords: Fashion game, Conformists, Rebels, Utility, t-satisfiability problem, Defective coloring.