Sign patterns of orthogonal matrices

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The sign-pattern of a real matrix $A = [a_{ij}]$ is the matrix obtained by replacing each of its positive entries by 1 and its negative entries by -1. In 1963, at one of the early international conferences in combinatorics, Miroslav Fielder posed the problem: Characterize the sign-patterns of $n \times n$ orthogonal matrices. No major progress on the problem occurred until the work of C. Johnson and C. Waters which provides strong necessary conditions for a sign-pattern to be that of an orthogonal matrix. In this talk we give a new proof of the Johnson-Waters result, which we conjecture to be the 'book proof', and describe a new technique, based on the implicit function theorem, that provides a way of producing many examples of sign-patterns of orthogonal matrices from one orthogonal matrix. The technique gives rise to interesting problems that involve solutions to matrix equations by matrices with given combinatorial patterns. Resolution of these problems involves a nice mixture of new and old graph theory and matrix theory.

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