Permutation binomials of index $q^{e-1}+\cdots+q+1$ over $\mathbb{F}_{q^e}$

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A binomial of the form $x^r(x^{q-1}+a)$ where $a \in \mathbb{F}_{q^e}^*$ and $e \geq 2$ is of index $q^{e-1}+\cdots+q+1$. In this work, we present several existence and nonexistence results for permutation binomials over $\mathbb{F}_{q^e}$ of this type. As a consequence, we obtain a complete characterization of such permutation binomials over $\mathbb{F}_{q^2}$, $\mathbb{F}_{q^3}$, $\mathbb{F}_{q^4}$, $\mathbb{F}_{p^5}$, and $\mathbb{F}_{p^6}$ where p is an odd prime. This extends previous results obtained by Li et al. and Liu for $e = 2$ and $e = 3$ with odd q, respectively.