

# Nonexistence of a Subfamily of a Family of Edge-Regular Graphs

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For  $d > 0$ , a simple  $d$ -regular graph  $G = (V, E)$  is *edge-regular* if there is some  $\lambda \in \mathbf{Z}$  such that every two adjacent vertices have precisely  $\lambda$  common neighbors. It is known that for  $d > \lambda > 0$ , the order  $|V| = n$  of such an edge-regular graph satisfies the inequality  $n \geq 3(d - \lambda)$ . It has also been shown that the extremal graphs for this inequality have the property for every adjacent pair  $u, v \in V$ , the subgraph induced by the intersection of their neighbor sets  $N(u) \cap N(v)$  is edgeless; and the extremal graphs have been identified. Some of the “next” families of edge-regular graphs of the inequality have been studied, such as the family of edge-regular graphs with parameters satisfying  $n = 3(d - \lambda) + 1$ . One of the interesting subfamilies of this family was when the common neighbor set of every adjacent vertex pair induced a disjoint union of  $K_2$ 's. We consider here the subfamily of the family of edge-regular graphs satisfying  $n = 3(d - \lambda) + 2$  with this interesting property.

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