Differences of Functions with the Same Value Set

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In a recent article, Ullman and Velleman studied functions $\mathbf{a}$ from an abelian group G to itself that can be expressed as a difference of two bijections $\mathbf{b}, \mathbf{c}$ from G to itself, and presented connections to directed graphs, juggling sequences, bus scheduling and Latin Squares. In this work we relax the condition that $\mathbf{b}$ and $\mathbf{c}$ are bijections and instead study which functions can be expressed as the difference of two functions with the same value set considered as a *multiset*. We characterize sequences $\mathbf{a} : G \longrightarrow G$ that can be a expressed in such a way. Given a function $\mathbf{a}$, we construct all possible $\mathbf{b}$, $\mathbf{c}$ with the same value set as multiset and such that $\mathbf{a} = \mathbf{b} - \mathbf{c}$.

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