Finite edge-transitive Cayley graphs, quotient graphs and Frattini groups

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The edge-transitivity of a Cayley graph is most easily recognisable if the subgroup of `affine maps' preserving the graph structure is itself edge-transitive. These are the so-called normal edge-transitive Cayley graphs. Each of them determines a set of quotients which are themselves normal edge-transitive Cayley graphs, and which are built from a very restricted family of groups (direct products of simple groups). We address the questions: how much information about the original Cayley graph can we retrieve from this special set of quotients? Can we ever reconstruct the original Cayley graph from them: if so, then how?

Our answers to these questions involve a type of `relative Frattini subgroup' determined by the Cayley graph, which has similar properties to the Frattini subgroup of a finite group. I'll discuss this and give some examples. It raises many new questions about Cayley graphs.

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