Knotted and Unknotted Eulerian Circuits on Graphs Embedded in Surfaces

Abdulmelik Mohammed*, Masahico Saito, Nataša Jonoska, University of South Florida

Graph theory has recently emerged as a powerful framework for the automated design of biomolecular nanostructures. A prime example of this is in the design of wireframe DNA origami nanostructures, where the routing of a circular viral DNA, called a scaffold strand, is modeled as an Eulerian circuit of a reconditioned triangulated mesh. In this setting, the knot type of the scaffold strand dictates the feasibility of an Eulerian circuit to be used as the scaffold route in the design. Motivated by these scaffold routings in 3D DNA structures, we investigate the knottedness of Eulerian circuits on surface-embedded graphs. We show that certain graph embeddings, called checkerboard colorable, always admit unknotted Eulerian circuits. On the other hand, we prove that if a graph admits an embedding in a torus such that the embedding is not checkerboard colorable, then the graph can be re-embedded so that all its non-intersecting Eulerian circuits are knotted. For surfaces of genus greater than one, we present an infinite family of checkerboard-colorable graph embeddings where there exist knotted Eulerian circuits.

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