Orthogonal symmetric matrices on the join of two graphs

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We introduce a combinatorial notion of compatibility for multiplicity matrices. This gives rise to a necessary condition for the join $G \vee H$ of two (possibly disconnected) graphs G and H to be the pattern of an orthogonal symmetric matrix, or equivalently, for the minimum number of distinct eigenvalues q of $G \vee H$ to be equal to two. Under additional technical hypotheses, this necessary condition is also sufficient.
This talk covers the first half of the preprint arXiv:2012.12694, and includes background for the related talks of Polona Oblak and Helena Šmigoc.

Keywords: Join of graphs; Inverse eigenvalue problem; Minimal number of distinct eigenvalues