On Local Antimagic Chromatic Number of Spider Graphs

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An edge labeling of a connected graph $G = (V, E)$ is said to be local antimagic if it is a bijection $f : E \to \{1, ..., |E|\}$ such that for any pair of adjacent vertices x and y, $f^+(x) \neq f^+(y)$, where the induced vertex label $f^+(x) = \sum f(e)$, with e ranging over all the edges incident to x. The local antimagic chromatic number of G, denoted by $\chi_{la}(G)$, is the minimum number of distinct induced vertex labels over all local antimagic labelings of G. In this paper, we first show that a d-leg spider graph has $d+1 \leq \chi_{la} \leq d+2$. We then obtain many sufficient conditions such that both the values are attainable. Finally, we show that each 3-leg spider has $\chi_{la} = 4$ if not all legs are of odd length. No 3-leg spider with all odd leg lengths and $\chi_{la} = 5$ is found. This provides partial solutions to the characterization of k-pendant trees T with $\chi_{la}(T) = k + 1$ or $k + 2$. We conjecture that almost all d-leg spiders of size q that satisfy $d(d + 1) \leq 2(2q - 1)$ with each leg length at least 2 has $\chi_{la} = d + 1$.

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