Zero forcing parameters, the ordered multiplicity inverse eigenvalue sequence problem for graphs and powers of graphs

Franklin H. J Kenter*, United States Naval Academy and Jephian C.-H. Lin, National Sun Yat-sen University

Given a graph G, one may ask: "What sets of eigenvalues are possible over all weighted adjacency matrices of G?" (Here, negative and diagonal weights are allowed). This is known as the Inverse Eigenvalue Problem for Graphs (IEPG) A mild relaxation of this question considers the multiplicity sequence instead of the exact eigenvalues themselves. For instance, given a graph G on n vertices and an ordered partition $\mathbf{m} = (m_1, \ldots, m_\ell)$ of n, is there a weighted adjacency matrix where the i-th distinct eigenvalue has multiplicity m_i? This is known as the ordered multiplicity inverse eigenvalue sequence problem. Recent work has solved this problem for all graphs on 6 vertices.
In this talk, we develop zero forcing methods for the ordered multiplicity IEPG in a multitude of different contexts. Namely, we utilize zero forcing parameters on powers of graphs to achieve bounds on sums of various multiplicities. Not only can we verify the above result in a more straight-forward manner, but we apply our techniques to skew-symmetric matrices, nonnegative matrices, among others.

Keywords: Zero forcing, minimum rank of graphs, combinatorial matrix theory, inverse eigenvalue problem