Cospanning Characterizations of Combinatorial Structures

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Given a finite set E and an operator $\sigma : 2^E \longrightarrow 2^E$, two sets $X, Y \subseteq E$ are *cospanning* if $\sigma(X) = \sigma(Y)$. Corresponding *cospanning equivalence relations* were investigated for greedoids in much detail (Korte, Lovasz, Schrader; 1991). For instance, these relations determine greedoids uniquely.
A notion of *violator space* was introduced in (Gärtner, Matoušek, Rüst, Škovroňby; 2008) as a combinatorial framework that encompasses linear programming and other geometric optimization problems.
Violator spaces are defined by violator operators. We prove that violator operators generalize rank closure operators of greedoids. Further, we introduce *co-violator spaces* based on contracting operators known also as choice functions. Let $\alpha, \beta : 2^E \longrightarrow 2^E$ be a violator operator and a co-violator operator, respectively. Cospanning characterizations of violator spaces allow us to obtain some new properties of these operators and their interconnections. In particular, we show that *uniquely generated* violator spaces enjoy so-called Krein-Milman properties, i.e., $\alpha(\beta(X)) = \alpha(X)$ and $\beta(\alpha(X)) = \beta(X)$ for every $X \subseteq E$.

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