Non-Singular cubic surfaces over \mathbb{F}_{2^k}

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We perform an opportunistic search for cubic surfaces over small fields of characteristic two. The starting point of our work is a list of surfaces complied by Dickson over the field with two elements. We consider the non-singular ones arising in Dickson's work for the fields of larger orders of characteristic two. We investigate the properties such as the number of lines, singularities and automorphism groups.

The problem of determining the possible numbers of lines of a non-singular cubic surface over the fields of $\mathbb{C}, \mathbb{R}, \mathbb{Q}, \mathbb{F}_q$ where q odd, \mathbb{F}_2 was considered by Cayley & Salmon, Schläfli, Segre, Rosati and Dickson, respectively. As the work in this paper will show various cases are overlooked by various authors. For instance, both Dickson and Segre overlooked the case of non-singular cubic surfaces with 4 lines in characteristic two. Besides that we investigate the structure of non-singular surfaces with 15 and 9 lines.

Keywords: Geometry, finite field, cubic surface, number of lines, Dickson surfaces.