Immersions of Cliques in Graph Products

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A graph G has a K_t-immersion if G contains a set of t vertices, every pair of which are connected by edge-disjoint paths. We define the *immersion number* of a graph G, denoted $\mathrm{im}(G)$, to be the largest value t for which G has a K_t-immersion. In this talk we will focus on clique immersions in graph products. In particular we ask, if G and H are graphs with $\mathrm{im}(G) = t$ and $\mathrm{im}(H) = r$ and $G * H$ is a particular graph product of G and H, how large is $\mathrm{im}(G * H)$? We will discuss the four standard graph products (Cartesian, lexicographic, direct, and strong products). Best possible lower bounds will be provided for the Cartesian and lexicographic products and conjectures will be offered for the direct and strong products.

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