

Conditions for Efficient (j, k) -Domination

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For a graph G , a function $f : V(G) \rightarrow \{0, 1, \dots, j\}$ is an *efficient (j, k) -dominating function* if, for all $v \in V(G)$,

$$\sum_{w \in N[v]} f(w) = k,$$

where $N[v]$ is the closed neighborhood of v (R.R. Rubalcaba and P.J. Slater. *Discussiones Mathematicae Graph Theory*, 2007). This definition immediately generalizes k -dominating sets. We characterize efficient (j, k) -dominating functions on strong products of graphs, determining the dominating functions on chess graphs on several surfaces as an application of our results. We also provide several necessary and sufficient conditions for the existence of such functions, characterizing all non-efficiently dominatable graphs of small order.

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