Minimum rank bounds for matrix patterns and graphs

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How does the placement of the nonzero entries in an $m \times n$ matrix constrain its rank? An important lower bound on the rank is the *triangle number* of the zero-nonzero pattern. How does the placement of the edges in a simple graph on n vertices constrain the rank of a symmetric $n \times n$ matrix whose off-diagonal nonzero entries occur in the same spots as those in the adjacency matrix of the graph? Here, a corresponding lower bound on the rank is (given by n minus) the *zero forcing number* of the graph. An important problem in both cases is that of when these lower bounds are met. Here we explore how to connect this problem in one scenario with the other. In particular, we exhibit a class of graphs with corresponding patterns for which the problems coincide. This allows us to give a graph whose maximum nullity is not equal to its zero forcing number over any field.

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