Dimensional k-Wiener index of a k-plex

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Motivated from the concepts of a k-*walk* and a k-*path* between two $(k-1)$-cells in a k-plex introduced by Beineke and Pippert in 1971, we introduce a new concept called the dimensional k-Wiener index of a k-plex as the summation of dimensional k-distances of every unordered pairs of $(k-1)$-cells of the k-plex. This is a generalization of the well-known concept of Wiener index from 1-dimensional graphs to k-plexes. The concept is different from the Wiener index of a hypergraph, which is the sum of distances between every unordered pair of vertices of the hypergraph. In this talk, we will focus on dimensional k-Wiener indices of k-trees, which form an important class of k-plexes and have many properties parallel to those of trees. We show that the dimensional k-Wiener indices of k-trees of order n are bounded below by $2\binom{1+(n-k)k}{2} - (n-k)\binom{k+1}{2}$ and above by $k^2\binom{n-k+2}{3} - (n-k)\binom{k}{2}$. We characterize the extremal graphs for the bounds as k-stars and a k-th power of paths, respectively. Our results generalize the well-known result that the Wiener indices of trees of order n are bounded between $(n-1)^2$ and $\binom{n+1}{3}$, and the bounds are attained only by stars and paths, respectively.

Keywords: k-plex, k-tree, dimensional k-Wiener index