Harmonious Colorings of Graphs

Alyssa Adams, Olivia Bindas, Erica Bajo Calderon, Madeline Cope, Alexis Byers*, Andrew Summers, and Rabin Thapa (Youngstown State University); Mohra Zayed (King Khalid University)

A harmonious labeling of a graph G of size m is an injective function $f : V(G) \to \mathbb{Z}_m$ that induces an injective function $f' : E(G) \to \mathbb{Z}_m$ defined by $f'(uv) = [f(u) + f(v)]$ (mod m). When G is a tree, then f is allowed to repeat one vertex label. We introduce a new type of graph labeling that combines this well-known concept with that of another, graph colorings. A proper vertex coloring $c : V(G) \to \mathbb{Z}_k$ is called a harmonious k-coloring if the induced edge coloring $c' : E(G) \to \mathbb{Z}_k$ defined by $c'(uv) = [c(u) + c(v)]$ (mod k) is also proper. The minimum positive integer k for which G has a harmonious k-coloring is the harmonious chromatic number of G, $\chi_h(G)$. We present our results in this area which include the harmonious chromatic number of all trees, cycles, and grids.

Keywords: harmonious labeling, proper vertex coloring, proper edge coloring