Expressing graphs as symmetric differences of cliques of the complete graph

Calum Buchanan*, University of Vermont; Chris Purcell, University of West Bohemia; Puck Rombach, University of Vermont

Any finite simple graph G can be represented by a collection $\mathscr{C}$ of subsets its vertex set so that two vertices appear in an odd number of sets in $\mathscr{C}$ if and only if they are connected by an edge in G. The minimum cardinality of such a collection is called the clique-build number of G, denoted $c_2(G)$. Such a collection induces a certain vector representation of G over the field of order 2, known as a faithful orthogonal representation. We explore the close relationship between $c_2(G)$, the minimum dimension of a faithful orthogonal representation of G over $\mathbb{F}_2$, and the minimum rank of G over $\mathbb{F}_2$. In particular, we show that $\mathrm{mr}(G, \mathbb{F}_2) \leq c_2(G) \leq \mathrm{mr}(G, \mathbb{F}_2) + 1$. In the case that G is a forest, we show equality of the clique-build number and the minimum rank of G. We provide upper bounds for $c_2(G)$ in terms of the number of vertices, the number of edges, and the minimum size of a vertex cover of G. Finally, we show that the graph property $c_2(G) \leq k$ is hereditary and finitely defined. We exhibit the sets of minimal forbidden induced subgraphs for $k = 1$ and $k = 2$.