

**The number of graph homomorphisms between cycles and cycles with loops,
A problem from Stanley's 1986 text *Enumerative Combinatorics***

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Let $g_k(n)$ be the number of sequences t_1, \dots, t_n in $\{0, 1, \dots, k-1\}$ such that $t_{j+1} \equiv t_j - 1, t_j$ or $t_j + 1 \pmod{k}$, $1 \leq j \leq n$, (where t_{n+1} is interpreted as t_1). It is proved combinatorially that $g_4(n) = 3^n + 2 + (-1)^n$ and $g_6(n) = 3^n + 2^{n+1} + (-1)^n$. This solves a problem from Richard P. Stanley's 1986 text, *Enumerative Combinatorics*.

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