Ulam Decompositions in Sparse Graphs

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Given two graphs G and H, each with n vertices and m edges, each graph edge set could be decomposed into r parts $E_G = E_{G_1} \cup E_{G_2} \cup ... \cup E_{G_r}$ and $E_H = E_{H_1} \cup E_{H_2} \cup ... \cup E_{H_r}$ such that G_i and H_i are isomorphic, this decomposition is called Ulam Decomposition and the minimum value of r is defined as $U(X, Y)$. Fan Chung, Ron Graham, Paul Erdős, Stan Ulam and Frances Yao did a lot of contributions to prove the bounds of $U(X, Y)$ and generalized it to multiple pairs or even infinite pairs of graphs and hypergraphs.
Given two graphs G and H, the determination of whether $U(G, H) \leq k$ is an NP-complete problem, even when $r = 2$, it is still NP-complete. In this paper we apply first-order logic and structural graph theory tools to show the complexity result of Ulam decompositions in Sparse Graphs.

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