Cut-edges and Regular Subgraphs in Odd-degree Regular Graphs

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Hanson, Loten, and Toft proved that every $(2r+1)$-regular graph with at most $2r$ cut-edges has a 2-factor. We generalize this: For $k \leq (2r+1)/3$, every $(2r+1)$-regular graph with at most $2r-3(k-1)$ cut-edges has a $2k$-factor. The restrictions on k and on the number of cut-edges are sharp. We characterize the graphs with exactly $2r-3(k-1)+1$ cut-edges but no $2k$-factor. For $k > (2r+1)/3$, there are graphs without cut-edges that have no $2k$-factor. (Joint work with Alexandr V. Kostochka, André Raspaud, Bjarne Toft, and Dara Zirlin.)

We prove that every 3-regular graph with c cut-edges has a 2-regular subgraph omitting at most $\lfloor (c-1)/2 \rfloor$ vertices (when $c > 0$), and this is sharp. The proof is by showing a more general result: Every multigraph having maximum degree 3, m edges, and exactly c cut-edges has a 2-regular subgraph that omits at most $\max\{0, \lfloor (3n-2m+c-1)/2 \rfloor\}$ vertices. The bound is sharp; we describe the extremal multigraphs.
(Joint work with Ilkyoo Choi, Ringi Kim, Alexandr V. Kostochka, and Boram Park.)

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