

Dimension and Height for Posets with Planar Cover Graphs

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For a positive integer h , let $d(h)$ be the largest integer t for which there exists a poset P of height h and dimension t such that the cover graph of P is planar. The fact that $d(h)$ is well defined was proved in 2014 by Streib and Trotter, but their extensive use of Ramsey theoretic techniques made it difficult to estimate how $d(h)$ grows in terms of h . Subsequently, stronger techniques produced better upper bounds on $d(h)$, and the best bound to date comes from a 2019 paper by Joret, Micek, Ossona de Mendez and Wiechert establishing connections between dimension and classes of nowhere dense graphs and weak coloring numbers. Their work shows that $\log(d(h)) = O(h^3)$, but this still allows that $d(h)$ could grow exponentially fast with h .

We extend concepts and techniques from both papers to prove that $d(h)$ is polynomial in h . Specifically, we show that $d(h) = O(h^6)$.

Keywords: planar graph, poset, cover graph, height, dimension,