Vertices Deletions in Clustered Spanning Tree

Nili Guttmann-Beck, Rony Rozen and Michal Stern*, Academic College of Tel-Aviv Yaffo, Yaffo, Israel

Let $H =< G, \mathcal{S} >$ be a hypergraph, where $G = (V, E)$ is a complete undirected graph and $\mathcal{S}$ is a set of not necessarily disjoint clusters $S_i \subseteq V$. The Clustered Spanning Tree problem is to find a spanning tree of G which satisfies that each cluster induces a subtree, when it exists.
We consider hypergraphs with no feasible solution. For these hypergraphs we suggest a way to choose vertices to be deleted in order to gain feasibility, using a special layers graph which represents the clusters' intersections. This paper presents algorithms which find a possible list of vertices whose deletion creates a new hypergraph, which has a feasible solution tree, and construct the corresponding solution tree. When all intersections contain more than $\frac{|V|}{2}$ vertices, we prove that the given hypergraph has a feasible solution tree and no vertices deletions are required. For slender reduction graphs, where no intersection set contains another intersection set, our algorithms achieve a minimum cardinality list of vertices deletions. We also present algorithms which either ensure that after the vertices deletions, every vertex will stay in at least one cluster, or that every cluster will still contain at least one vertex.

Keywords: Clustered Spanning Tree, Feasibility, vertices deletions.