Game Proper Chromatic Numbers

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Consider the following two-person game on a graph G. The two players start with two color choices only, taking turns coloring any uncolored vertex with the restriction that any coloring must be a proper coloring. A third (or forth, etc.) color can only be used when forced to maintain a proper coloring. One player, the minimizer, is trying to force the smallest number of colors possible. The other player, the maximizer, is trying to force the largest number of colors possible. The game chromatic number III, denoted $\chi_{(E,g)}(G)$, is the minimum number colors used when both players play optimally. The advantage of the game chromatic number III is that it is comparable to both the game chromatic number and the game chromatic number II while it is still unknown if the game Grundy number is comparable to any of these game chromatic numbers. Now, let $R = \{R_1, R_2, \ldots, R_t\}$ such that $\bigcup R_i = V(G)$. It is convenient to think of these R_i's as regions of interest in graph G. The game chromatic number III can be extended to any of these regions. This talk will consider extensions to closed neighborhoods and open neighborhoods maintaining the restriction that all colorings must be "proper" in the sense that no R_i is monochromatic. The minimum number of colors necessary provided each player plays optimally, following the rules established for the game chromatic number III, is denoted $\chi_{(N[v],g)}(G)$ and $\chi_{(N(v),g)}(G)$, respectively.

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