Subtractive Magic and Antimagic Total Labeling for Basic Families of Graphs

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A *subtractive arc-magic labeling* (SAML) of a directed graph $G = (V, A)$ is a bijection $\lambda : V \cup A \to \{1, 2, \ldots, |V| + |A|\}$ with the property that for every $xy \in A$ we have $\lambda(xy) + \lambda(y) - \lambda(x)$ equals to an integer constant. If $\lambda(xy) + \lambda(y) - \lambda(x)$ are distinct for every $xy \in A$, then λ is a *subtractive arc-antimagic labeling* (SAAL). A *subtractive vertex-magic labeling* (SVML) of G is such bijection with the property that for every $x \in V$ we have $\lambda(x) + \sum_{y \in V, yx \in A} \lambda(yx) - \sum_{y \in V, xy \in A} \lambda(xy)$ equals to an integer constant. If $\lambda(x) + \sum_{y \in V, yx \in A} \lambda(yx) - \sum_{y \in V, xy \in A} \lambda(xy)$ are distinct for every $x \in V$, then λ is a *subtractive vertex-antimagic labeling* (SVAL). In this paper we prove some existence or non-existence of SAML, SVML, SAAL, and SVAL for several basic families of directed graphs, such as paths, cycles, stars, wheels, tadpoles, friendship graphs, and general butterfly graphs. The constructions are given when such labeling(s) exists.

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