Gallai Ramsey numbers of complete graphs and odd cycles

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Given a graph H, the k-coloured Gallai Ramsey number $gr_k(K_3 : H)$ is defined to be the minimum integer n such that every k-colouring (using all k colours) of the complete graph on n vertices contains either a rainbow triangle or a monochromatic copy of H. In 2015, Fox, Grinshpun, and Pach conjectured the value of the Gallai Ramsey numbers for complete graphs. We verify this conjecture for the first open case when $H = K_4$. For the case $H = K_5$ we will show that the validity of the conjecture depends on the exact value of the (unknown) Ramsey number $r(K_5, K_5)$. We also present the Gallai Ramsey numbers $gr_k(K_3 : C_{2p+1})$ for all odd cycles C_{2p+1} .

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