Solution of an Infinite Nested Recurrence Relation

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We determine a recursive structural solution to the infinite nested recurrence relation (with $a(n) = 0$ if $n \leq 0$)

$$a(n) = n-1-a(n-1)-a(a(n-2))-a(a(a(n-3)))-a(a(a(a(n-4))))-\cdots.$$

This structure shows that $a(n)$ takes on Fibonacci values at Fibonacci arguments. We make use of a general result relating certain morphisms and the solution of a class of nested recurrence relations.

Keywords: nested recurrence relations, Fibonacci numbers, morphism.