**The normalized distance Laplacian Matrix**

Carolyn Reinhart, Iowa State University

The distance matrix $\mathcal{D}(G)$ of a graph $G$ is the matrix containing the pairwise distances between vertices. The transmission of a vertex $v_i$ in $G$ is the sum of the distances from $v_i$ to all other vertices and $T(G)$ is the diagonal matrix of transmissions of the vertices of the graph. The normalized distance Laplacian $\mathcal{D}^{\mathcal{L}}(G) = I - T(G)^{-1/2}\mathcal{D}(G)T(G)^{-1/2}$ is introduced. This is analogous to the normalized Laplacian matrix, defined such that $\mathcal{L}(G) = I - D(G)^{-1/2}A(G)D(G)^{-1/2}$ where $D(G)$ is the diagonal matrix of degrees of the vertices of the graph and $A(G)$ is the adjacency matrix. Bounds on the spectral radius of $\mathcal{D}^{\mathcal{L}}$ and connections with the normalized Laplacian matrix will be presented. The generalized distance characteristic polynomial will be defined and its properties discussed. Finally, methods for determining eigenvalues of $\mathcal{D}^{\mathcal{L}}$ will be discussed, including the use of twin vertices.

Keywords: spectral graph theory, distance matrix, normalized Laplacian