Forbidden induced subgraphs and Gallai vertices

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A Gallai set is a set of vertices that intersects every longest path in G, and a Gallai vertex is a vertex that forms a Gallai set of size 1. Let \mathcal{H} be the set of graphs H such that every connected H-free graph has a Gallai vertex. Golan and Shan recently proved that $2K_2 \in \mathcal{H}$. We show that every graph in \mathcal{H} is a linear forest on at most 9 vertices, and this is sufficient when $|V(H)| \leq 4$. We also show that large, dense graphs have sublinear Gallai sets.

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