An Ordered Tuple Construction of Classical Geometric (Clifford) Algebras and Geometric Algebras with Signature.

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Geometric or Clifford algebras are an increasingly useful structure for studying graphs. In a previous talk we constructed the geometric algebra $\mathbb{G}^3$ by defining a geometric product on ordered 8-tuples in a manner that resembles the ordered four-tuple construction of the quaternions. In this talk we will use enumeration and properties of finite dimensional algebraic structures to construct *any* classical Clifford Algebra $\mathbb{G}^n$ by similarly defining a geometric product on $\mathbb{R}^{(2^n)}$. We will establish a basis of monomials for $\mathbb{G}^n$ by multiplying those tuples which generate $\mathbb{G}^n$; specifically the images canonically mapped from any orthonormal basis on $\mathbb{R}^n$.

By similarly embedding an orthonormal basis from $\mathbb{R}^n$ equipped with a quadratic form, we will extend this construction to the Clifford algebra with signature (p, q) where $p + q = n$, denoted $\mathbb{G}^{p,q}$. For specified values of n, we will derive a new set of generators for $\mathbb{G}^{0,n}$ that define a Clifford graph algebra, and show how these algebras can represent path graphs and star graphs.

Keywords : Clifford algebra, geometric product, path graph, star graph.