The maximum p-Spectral Radius of Hypergraphs with m Edges

Linyuan Lu, University of South Carolina

For $r \geq 2$ and $p \geq 1$, the p-spectral radius of an r-uniform hypergraph $H = (V, E)$ on n vertices is defined to be

$$\rho_p(H) = \max_{\mathbf{x}\in\mathbb{R}^n:\|\mathbf{x}\|_p=1} r \cdot \sum_{\{i_1,i_2,\ldots,i_r\}\in E(H)} x_{i_1}x_{i_2}\cdots x_{i_r},$$

where the maximum is taken over all $\mathbf{x} \in \mathbb{R}^{\mathbf{n}}$ with the p-norm equals 1. In this paper, we proved for any integer $r \geq 2$, and any real $p \geq 1$, and any r-uniform hypergraph H with $m = \binom{s}{r}$ edges (for some real $s \geq r-1$), we have

$$\lambda_p(H) \leq \frac{rm}{s^{r/p}}.$$

The equality holds if and only if s is an integer and H is the complete r-uniform hypergraph K_s^r with some possible isolated vertices added. Thus, we completely settled a conjecture of Nikiforov. In particular, we settled all the principal cases of the Frankl-Füredi's Conjecture on the Lagrangians of r-uniform hypergraphs for all $r \geq 2$.

Keywords: p-Spectral Radius, Langragian, Hypergraphs