Diagonalizable matrices and cycle structures of their digraphs

Xinlei Feng, Leshan Normal University; Frank J. Hall, Guangming Jing, Zhongshan (Jason) Li*, Chris Zagrodny, Georgia State University; Jiang Zhou, Harbin Engineering University.

A sign pattern (matrix) is a matrix whose entries are from the set $\{+, -, 0\}$. A square sign pattern $\mathcal{A}$ is said to allow diagonalization if there is a diagonalizable real matrix whose entries have signs specified by the corresponding entries of $\mathcal{A}$. It is known that for every sign pattern that allows diagonalization, its maximum composite cycle length is greater than or equal to its minimum rank. It is also known that a sign pattern allows diagonalization if and only if it allows rank-principality. Characterization of sign patterns that allow diagonalization has been a long-standing open problem. In this talk, we establish some new necessary/sufficient conditions for a sign pattern to allow diagonalization, and explore possible ranks of diagonalizable matrices with a specified sign pattern. In particular, it is shown that every irreducible sign pattern with minimum rank 2 allows diagonalization at rank 2 and also at the maximum rank. Sign patterns whose maximal zero submatrices are "strongly disjoint" are shown to have a composite cycle consisting of 1-cycles, 2-cycles, and at most one 3-cycle, with total length equal to the maximum rank; for such sign patterns, the maximum composite cycle length is invariant under row and column permutations.

Keywords: Sign pattern matrix, allowing diagonalization, rank-principality, maximum composite cycle length