W_2-graphs and shedding vertices

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A graph is *well-covered* if all its maximal independent sets are of the same size (Plummer, 1970). A well-covered graph is 1-*well-covered* if the deletion of every vertex leaves a graph, which is well-covered as well (Staples, 1975).
A graph G belongs to class W_n if every n pairwise disjoint independent sets in G are included in n pairwise disjoint maximum independent sets (Staples, 1975). Clearly, W_1 is the family of all well-covered graphs. It turns out that $G \in \mathbf{W_2}$ if and only if it is a 1-well-covered graph without isolated vertices.
For a graph G, let us define $\varepsilon_G : \mathrm{Ind}(G) \longrightarrow \mathbb{N}$ as $\varepsilon_G(A) = \max\{|S| : A \subseteq S \text{ and } S \in \mathrm{Ind}(G)\}$, where $\mathrm{Ind}(G)$ is the family of all the independent sets. The function ε_G may be interpreted as the strength of enlargement of independent sets. We describe shedding vertices as the ones that have no impact on values of ε_G, i.e., $v \in \mathrm{Shed}(G)$ if and only if $\varepsilon_{G-v}(A) = \varepsilon_G(A)$ for each $A \in \mathrm{Ind}(G-v)$. Specifically, for well-covered graphs, it means that the vertex v is shedding if and only if $G-v$ is well-covered.

Keywords: maximum independent set, shedding vertex, matching, 1-well-covered graph