Edge-colorings of hypergraphs avoiding a rainbow expanded complete graph

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For integers $\ell \geq k \geq 2$, the *expanded complete graph* $H^{(k)}_{\ell+1}$ is the k-uniform hypergraph obtained from the complete graph $K_{\ell+1}$ on $(\ell+1)$ vertices by enlarging every edge by $(k-2)$ new vertices. Pikhurko has solved the Turán problem for $H^{(k)}_{\ell+1}$: the unique extremal n-vertex k-uniform hypergraph for $H^{(k)}_{\ell+1}$ is the *balanced, ℓ-partite, complete, k-uniform (Turán) hypergraph* $T^{(k)}_{\ell}(n)$.
Here we consider a hypergraph version of an edge-coloring problem first considered by Erdős and Rothschild for graphs. For integers r and n, and a fixed graph F, the original problem concerned n-vertex graphs with the largest number of r-edge colorings that do not contain a monochromatic copy of F. An *r-pattern* P of a k-uniform hypergraph F is a partition its set of hyperedges into at most r classes. An r-coloring of a k-uniform hypergraph $H = (V, E)$ is *(F, P)-free* if H does not contain a copy of F for which the partition of its set of hyperedges induced by the coloring is isomorphic to P. The aim is to characterize those n-vertex k-uniform hypergraphs H with the largest number of (F, P)-free r-colorings.
For F being a complete graph and the *monochromatic pattern*, Alon, Balogh, Keevash and Sudakov showed for $r \in \{2, 3\}$ colors, that the (Turán) F-extremal graph is the only extremal graph for the edge-coloring problem. This is also the case here, where we consider $F = H^{(k)}_{\ell+1}$ and the *rainbow pattern*, where $r \geq r_0(k, \ell)$. This is joint work with Lucas de Oliveira Contiero, Carlos Hoppen and Knut Odermann.

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