Wiener index on unicyclic graphs

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The Wiener index W(G) of a graph G is defined as the sum of the lengths of the shortest paths between all unordered pairs of vertices in G. This graph parameter, also known as gross status or transmission, is a widely studied topological index of graphs with rich applications. Yet, there are still many unsolved problems, even on such simple class as unicyclic graphs.

In the first part of the talk, I will speak about an open problem posed by Du in [Wiener indices of trees and monocyclic graphs with given bipartition. International Journal of Quantum Chemistry, 112:15981605, 2012]. As a solution to this problem, we determine the unicyclic bipartite graphs with given size of parts having the maximum Wiener index.

The second part of the talk will be devoted to *Šoltés problem*. The problem is to find all graphs G such that the equality W(G) = W(G - v) holds for all their vertices $v \in V(G)$. The only known graph with this property is C_{11} . We provide a related result: For any given $k \in N$, there exists a unicyclic graph such that there are exactly k vertices preserving its Wiener index upon removal. This solves an open problem posed by Knor, Majstorovic and Skrekovski.

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