Interval Orders with Length Bounds

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We review some basic tools for looking at interval orders with side conditions such a bounds on the interval lengths. These make use of systems of linear inequalities and negative cycles in weighted digraphs. We use this approach to provide forbidden suborder characterizations in two different settings. For one, the interval for each element of the order has a prespecified length of 1 or 2. The second refines work by Peter Fishburn around the time of CGTC 15. He provided conditions for an order to have an interval representation with lengths between p and q. When $p = 1$ an order with no such representation contains a $2 + 2$ (not an interval order) or a $(1 + (q + 2))$. The characterization for $p \geq 2$ did not yield a list of minimal forbidden orders. We provide a list of minimal orders that block such a representation when the lower length bound is 2 or 3 and refine the original conditions in the general case.