The Pseudograph and Simple Graph Threshold Number for Bounded Factorizations

Anthony Hilton, University of Reading

For $d \geq 1$, $s \geq 0$ a $(d, d+s)$-*graph* is a graph whose degrees all lie in the interval $\{d, d+1, \ldots, d+s\}$. For $r \geq 1$, $a \geq 0$, an $(r, r+a)$-*factor* of a graph G is a spanning $(r, r+a)$-subgraph of G. An $(r, r+a)$-*factorization* of a graph G is a decomposition of G into edge-disjoint $(r, r+a)$-factors. A graph is $(r, r+a)$-*factorable* if it has an $(r, r+a)$-factorization. Let $\sigma(r, s, a, t)$ be the least integer such that, if $d \geq \sigma(r, s, a, t)$, then every $(d, d+s)$-simple graph G is $(r, r+a)$-factorable with x factors for at least t different values of x.
In this paper we evaluate $\sigma(r, s, a, t)$ for all values of r, s, a and t. We also show that if $a \geq 2$ and $r \geq 1$, then, when r is even and a is odd, every $(d, d+s)$-simple graph G has an $(r, r+a)$-factorization with x factors if and only if

$$\frac{d+s}{r+a} < x \leq \frac{d}{r},$$

and we prove similar statements for other parities of r and a.
We also evaluate $\pi(r, s, a, t)$, which is the threshold function for pseudographs (where multiple edges and multiloops are allowed).
This is joint work with Anitha Rajkumar.

Keywords: Factorizations, Bounded Degree Factorizations, Simple Graphs and Pseudographs