Partitioning a cograph into forests and stable sets

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Cographs are graphs not containing P_4 as an induced subgraph. It was proved by Damaschke in 1990 that every hereditary property is characterized by finitely many forbidden induced subgraphs (minimal obstructions) in the class of cographs. Cographs are perfect, and thus, K_k is the only minimal obstruction to k-colouring (partition into k stable sets). A natural way of generalizing k-colouring is to consider vertex partitions into k sparse sets. Depending on the definition of sparsity, we will have different vertex partition problems. Stable sets are the sparsest possible sets, and they correspond to the k-colouring problem. If we consider forests, we have the vertex arboricity problem: a graph has vertex arboricity k if its vertex set can be partitioned into at most k forests.
In this work, we exhibit the complete family of minimal obstructions for a cograph to have vertex arboricity 2. Also, we consider the problem of partitioning a cograph into p forests and q stable sets; for $p = 1$ and $q = 1$, this problem corresponds to the independent feedback vertex set. We show that for any q, this problem has exactly two minimal obstructions. Some observations are made for the general case for both problems.

Keywords: cographs, vertex arboricity, feedback vertex set