

Ternary Strings and the Pell Numbers

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For $n \geq 1$ let a_n count the number of sequences $s_1, s_2, s_3, \dots, s_n$ where (i) $s_1 = 0$; (ii) $s_i \in \{0, 1, 2\}$, for $2 \leq i \leq n$; and, (iii) $|s_i - s_{i-1}| \leq 1$, for $2 \leq i \leq n$. Then $a_1 = 1$, $a_2 = 2$, $a_3 = 5$, $a_4 = 12$, and $a_5 = 29$. In general, for $n \geq 3$, $a_n = 2a_{n-1} + a_{n-2}$, and a_n equals P_n , the n th Pell number.

For these P_n sequences of length n , we count (i) the number of occurrences of each of the symbols 0, 1, 2; (ii) the number of times each of the symbols 0, 1, 2 occur in a given position; (iii) the number of levels, rises and descents that occur within the sequences; (iv) the number of runs that occur within the sequences; (v) the sum of all the sequences considered as base 3 integers; (vi) the number of inversions and coinversions for the sequences; and, (vii) the sum of the major indices for the sequences.

Finally, from the numbers of occurrences of each symbol in each of the n possible locations for the sequences of length n , we find an example of the hexagonal property.