Failed Power Domination on Knödel Graphs

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Let G be a simple graph with vertex set V and edge set E, and let $S \subseteq V$. The *open neighborhood* of $v \in V$, $N(v)$, is the set of vertices adjacent to v; the *closed neighborhood* is given by $N[v] = N(v) \cup \{v\}$. The *open neighborhood* of S, $N(S)$, is the union of the open neighborhoods of vertices in S, and the *closed neighborhood* of S is $N[S] = S \cup N(S)$. The sets $\mathcal{P}^i(S), i \geq 0$, of vertices *monitored* by S at the i^{th} step are given by $\mathcal{P}^0(S) = N[S]$ and $\mathcal{P}^{i+1}(S) = \mathcal{P}^i(S) \bigcup \{w : \{w\} = N[v] \backslash \mathcal{P}^i(S)$ for some $v \in \mathcal{P}^i(S)\}$. If there exists j such that $\mathcal{P}^j(S) = V$, then S is called a *power dominating set*, PDS, of G.

We introduce and discuss the *failed power domination number* of a graph G, $\bar{\gamma}_p(G)$, which is the cardinality of the largest subset of V that is not a PDS. We show cases where $\bar{\gamma}_p(G)=$ $|V|$ through $|V|-3$. Also shown are results for some named graphs, such as Knödel graphs.

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