The 5-cube Cut Number Problem: A Short Proof for a Basic Lemma

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The hypercube cut number $S(d)$ is the minimum number of hyperplanes in the d-dimensional Euclidean space $\mathbb{R}^d$ that slice all the edges of the d-cube. The problem originally was posed by P. O'Neil in 1971. B. Grünbaum, V. Klee, M. Saks and Z. Füredy have raised the problem in different times. In 2002, Sohler and Ziegler obtained a computational solution to the 5-cube problem. However finding a short proof for the problem, independent of computer computations, remains to be a challenging problem. We present a short proof for the result presented by Emamy-Uribe-Tomassini in Hypercube 2002 based on the Tomassini's Thesis. The proof here is substantially shorter than the original Tomassini's proof of 60 pages.

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