

Origami Knotting in Graphs

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We present a knotting problem in spatial graphs arising from DNA self-assembly that involves Euler circuits as opposed to the cycles of intrinsically linked and knotted graphs. Design strategies for the origami method of self-assembly use a single circular (unknotted) strand of DNA called the *scaffolding strand* which must traverse all the edges of the target graph, subject to constraints on how it may pass through the vertices and how/if any edges are repeated. If the target structure is modeled as a graph embedded on an oriented surface in 3-space, then these routes correspond to *A-trails*, which are Eulerian circuits that turn either left or right at each vertex. If the surface is a sphere, then all A-trails in the target structure are necessarily unknotted and so provide possible scaffolding strand routes. On higher-genus surfaces however there are settings in which every A-trail is knotted. For geometrically embedded graphs that are not necessarily surface meshes, the initial challenge is formalizing the constraints on how the scaffolding strand may pass through vertices. These are captured *O-trails*, which generalize A-trails and provide a general framework in which to study origami knotted graphs, that is, graphs with knotted O-trails.

Keywords: spatial graphs, knots, intrinsic knotting, graph embedding, DNA self-assembly, DNA origami, A-trail, O-trail