Quadrangular embeddings of complete graphs

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Triangular embeddings of complete graphs, where every face is bounded by a 3-cycle, played an important role in the proof of the Map Color Theorem, which gives a sharp bound on the chromatic number of graphs embeddable in a given surface other than the sphere. Quadrangular embeddings of complete graphs are those where every face is bounded by a 4-cycle, and also have an application to a map coloring result. We show that quadrangular or nearly quadrangular embeddings, both orientable and nonorientable, exist for every complete graph K_n with $n \geq 7$. We also describe the coloring application.

Keywords: quadrangular embedding, complete graph, map coloring, chromatic number