

## Stack-Sorting, Set Partitions, and Lassalle’s Sequence

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Around 2010, Zeilberger introduced a curious sequence  $(A_m)_{m \geq 1}$  that is defined via a recurrence relation involving alternating signs and Catalan numbers. This sequence is now called “Lassalle’s sequence” because Lassalle gave an algebraic proof that its terms are positive and increasing, settling a conjecture of Zeilberger. We exhibit a natural combinatorial interpretation of Lassalle’s sequence. Namely,  $A_{k+1}$  is the number of permutations of length  $2k + 1$  that have exactly one preimage under West’s stack-sorting map. The proof is a special consequence of a much more general bijection between recently-introduced objects called “valid hook configurations” and certain weighted set partitions that Josuat-Vergès studied in the context of free probability theory.

Keywords: Stack-sorting, set partition, Lassalle’s sequence, Tutte polynomial, free probability.