C_4-face-magic torus labelings on $C_{2m} \times C_{2n}$

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For a toroidal graph $G = (V, E)$ embedded in the torus, let $\mathcal{F}(G)$ denote the set of faces of G. Then, G is called a C_n-*face-magic torus graph* if there exists a bijection $f : V(G) \rightarrow \{1, 2, \ldots, |V(G)|\}$ such that for any $F \in \mathcal{F}(G)$ with $F \cong C_n$, the sum of all the vertex labelings along C_n is a constant S. Let $x_v = f(v)$ for all $v \in V(G)$. We call $\{x_v : v \in V(G)\}$ a C_n-*face magic torus labeling* on G. We investigate C_4-face-magic torus labelings on $C_{2m} \times C_{2n}$. We say that a C_4-face-magic torus labeling $\{x_{i,j}\}$ on $C_{2m} \times C_{2n}$ is *antipodal balanced* if $x_{i,j} + x_{i+m,j+n} = (1/2)S$ for all $(i, j) \in V(C_{2m} \times C_{2n})$. We show that there exists a C_4-face-magic antipodal balanced torus labeling on $C_{2m} \times C_{2n}$ for all positive integers m and n. We investigate properties of these labelings on $C_{2m} \times C_{2n}$. We also determine all C_4-face-magic antipodal balanced torus labelings on $C_4 \times C_4$ up to symmetries on a torus.

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